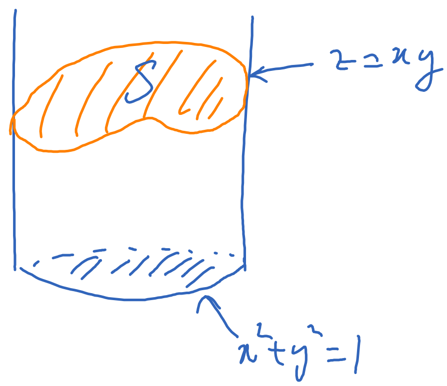
Example of finding surface area

Tuesday, April 6, 2021 4:03 PM

Recall: $\text{Area}(S) = \iint_D |r_u \times r_v| \, dA$

Ex



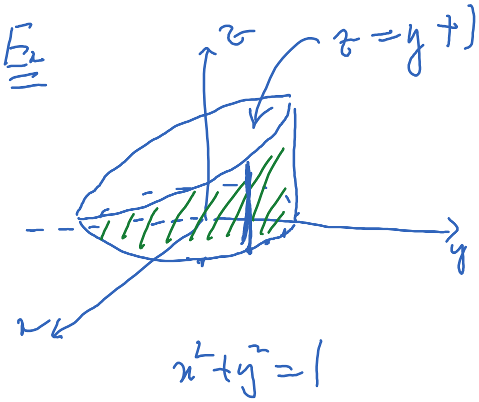
$r: \begin{cases} x = x \\ y = y \\ z = xy \end{cases}$ $(x,y) \in D$ ← unit disc

$r = \langle x, y, xy \rangle$

$r_x = \langle 1, 0, y \rangle$, $r_y = \langle 0, 1, x \rangle$ $|\langle -y, -x, 1 \rangle|$

$\text{Area} = \iint_D \sqrt{x^2+y^2+1} \, dA = \int_0^1 \int_0^{2\pi} \sqrt{r^2+1} \, r \, d\theta \, dr = \cdots$

Ex



$r: \begin{cases} x = \cos\theta \\ y = \sin\theta \\ z = z \end{cases}$ $\underbrace{\begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le z \le \sin\theta + 1 \end{matrix}}_{D}$

cut by the planes $z = y+1$ and $\underline{\underline{z = 0}}$

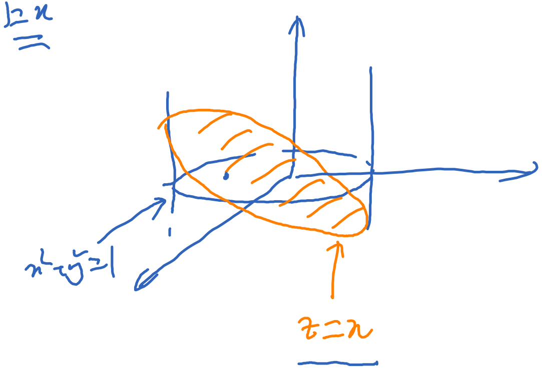
$\text{Area} = \iint_D |r_\theta \times r_z| \, dA = \iint_D \underbrace{|\langle -\sin\theta, \cos\theta, 0 \rangle \times \langle 0, 0, 1 \rangle|}_{\underbrace{|\langle \cos\theta, \sin\theta, 0 \rangle|}_{1}} \, dA = \text{area}(D)$

$\begin{matrix} -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{matrix}$

$\iint_D dA$

$= \int_0^{2\pi} \int_0^{\sin\theta + 1} dz \, d\theta = \cdots$

Ex



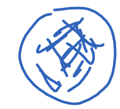
$f(x,y) = x$

Two ways;

$$R_1: \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = r\cos\theta \end{cases} \qquad \left.\begin{matrix} 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \end{matrix}\right]$$

$$R_2: \begin{cases} x = x \\ y = y \\ z = x \end{cases} \qquad (x,y) \in D$$

disc



$$\text{Area} = \int_0^{2\pi}\int_0^1 \left| \frac{\partial R_1}{\partial r} \times \frac{\partial R_1}{\partial \theta} \right| dr\, d\theta$$

$$= \iint_D \left| \overbrace{\frac{\partial R_2}{\partial x}}^{\langle 1,0,1\rangle} \times \overbrace{\frac{\partial R_2}{\partial y}}^{\langle 0,1,0\rangle} \right| dA = \int_0^{2\pi}\int_0^1 \left| \frac{\partial R_2}{\partial x} \times \frac{\partial R_2}{\partial y} \right| r\,dr\,d\theta$$